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Standard Practice for Calculating Thermal Transmission Properties Under Steady-State Conditions[1]

This standard is issued under the fixed designation C 1045; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice covers requirements and guidelines for the determination of thermal transmission properties based upon steady-state one dimensional heat transfer tests on a thermal insulation material or system for which values of heat flux, surface or air temperatures, and specimen geometry are reported from standard test methods.

1.2 The thermal transmission properties described include: thermal conductance, thermal resistance, apparent thermal conductivity, apparent thermal resistivity, surface conductance, surface resistance, and overall thermal resistance or transmittance.

1.3 This practice is restricted to calculation of thermal transmission properties from heat transfer data generated by standard test methods. These methods include: (1) planar geometries such as those used in Test Methods C 177, C 236, C 518, C 745, C 976, and C 1114, and (2) radial geometries such as those used in Test Methods C 335 and C 1033.

1.4 This practice includes the procedure for development of thermal conductivity as a function of temperature equation from data generated by standard test methods.

1.5 The values stated in SI units are to be regarded as the standard.

1.6 The attached appendixes provide discussions of the thermal properties of thermal insulating materials, the development of the basic relationships used in this practice, and examples of their use.

2. Referenced Documents

2.1 *ASTM Standards:*

C 168 Terminology Relating to Thermal Insulating Materials[2]

C 177 Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded-Hot-Plate Apparatus[2]

C 236 Test Method for Steady-State Thermal Performance of Building Assemblies by Means of a Guarded Hot Box[2]

C 335 Test Method for Steady-State Heat Transfer Properties of Horizontal Pipe Insulations[2]

C 518 Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Heat Flow Meter Apparatus[2]

C 680 Practice for Determination of Heat Gain or Loss and the Surface Temperature of Insulated Pipe and Equipment Surfaces by the Use of a Computer Program[2]

C 745 Test Method for Heat Flux Through Evacuated Insulations Using a Guarded Flat Plate Boiloff Calorimeter,[2]

C 976 Test Method for Steady-State Thermal Performance of Building Assemblies by Means of a Calibrated Hot Box[2]

C 1033 Test Method for Steady-State Heat Transfer Properties of Pipe Insulation Installed Vertically[2]

C 1058 Practice for Selecting Temperatures for Evaluating and Reporting Properties of Thermal Insulation[2]

C 1114 Test Method for Steady-State Thermal Transmission Properties by Means of the Thin-Heater Apparatus[2]

E 122 Practice for Choice of Sample Size to Estimate the Average Quality of a Lot or Process[3]

3. Terminology

3.1 *Definitions*—The definitions and terminology of this practice are intended to be consistent with Terminology C 168. However, because exact definitions are critical to the use of this practice, the following equations are defined here for use in the calculations section of this practice.

3.2 *Symbols*—The symbols, terms and units used in this practice are the following:

A	=	specimen area normal to heat flux direction, m^2,
λ	=	thermal conductivity or apparent thermal conductivity, W/(m · K),
$\lambda(T)$	=	the functional relationship between thermal conductivity and temperature, W/(m K),
λ_{exp}	=	the experimental thermal conductivity, W/(m K),
λ_m	=	mean thermal conductivity, averaged with respect to temperature from T_c to T_h, W/(m · K),
C	=	thermal conductance, W/(m^{2}K),
h_h	=	surface coefficient, hot side, W/(m^{2}K),
h_c	=	surface coefficient, cold side, W/(m^{2}K),
l	=	metering area length in the axial direction, *m*,

[1] This practice is under the jurisdiction of ASTM Committee C-16 on Thermal Insulation and is the direct responsibility of Subcommittee C16.30 on Thermal Measurements.

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[2] *Annual Book of ASTM Standards*, Vol 04.06.

[3] *Annual Book of ASTM Standards*, Vol 14.02.

q = one-dimensional heat flux (time rate of heat flow through metering area divided by the apparatus metering area A), W/m^2,
Q = time rate of one-dimensional heat flow through the metering area of the test apparatus, W,
r_i = radius of a hollow cylinder at the i th surface, m,
r' = thermal resistivity, K m/W,
R = thermal resistance, m^{2}K/W,
R_h = surface resistance, hot side, m^{2}K/W,
R_c = surface resistance, cold side, m^{2}K/W,
R_u = overall thermal resistance, m^{2}K/W,
T = temperature, K,
T_1 = area-weighted air temperature 75 mm or more from the hot side surface, K,
T_h = area-weighted temperature of specimen hot surface, K,
T_c = area-weighted temperature of the specimen cold surface, K,
T_2 = area-weighted air temperature 75 mm or more from the cold side surface, K,
T_m = specimen mean temperature, average of two opposite surface temperatures, $(T_h + T_c)/2$, K,
ΔT = temperature difference, K,
$\Delta T_{s\text{-}s}$ = temperature difference, surface to surface, $(T_h - T_c)$, K,
$\Delta T_{a\text{-}a}$ = temperature difference, air to air, $(T_1 - T_2)$, K,
U = thermal transmittance, W/(m^{2}K), and
x = linear dimension in the heat flow direction, m.

3.3 *Thermal Transmission Property Equations*:

3.3.1 *Thermal resistance*, R, is defined in Terminology C 168. It is not necessarily a unique function of temperature or material, but is rather a property determined by the specific thickness of the specimen and by the specific range of temperatures used to measure the thermal resistance.

$$R = \frac{A\,(T_h - T_c)}{Q} \tag{1}$$

3.3.2 *Thermal Conductance, C*:

$$C = \frac{Q}{A\,(T_h - T_c)} \tag{2}$$

NOTE 1—Thermal resistance, R, and the corresponding thermal conductance, C, are reciprocals, that is, their product is unity. These terms apply to specific bodies or constructions as used, either homogeneous or heterogeneous, between two specified isothermal surfaces.

NOTE 2—Subscripts h and c are used to differentiate between hot side and cold side surfaces.

3.3.3 *Apparent thermal conductivity*, λ, is defined in Terminology C 168.

Rectangular coordinates:

$$\lambda = \frac{Q\,L}{A\,(T_h - T_c)} \tag{3}$$

Cylindrical coordinates:

$$\lambda = \frac{Q\,\ln(r_2/r_1)}{2\,\pi\,l\,(T_h - T_c)} \tag{4}$$

where:
r_1 = inner radius,
T_h = temperature at the inner radius,
r_2 = outer radius, and
T_c = temperature at the outer radius.

3.3.4 *Apparent thermal resistivity*, r, is defined in Terminology C 168.

Rectangular coordinates:

$$r' = \frac{A\,(T_h - T_c)}{Q\,L} \tag{5}$$

Cylindrical coordinates:

$$r' = \frac{2\,\pi\,l\,(T_h - T_c)}{Q\,\ln\,(r_2\,/\,r_1)} \tag{6}$$

NOTE 3—Thermal resistivity, r', and the corresponding thermal conductivity, λ, are reciprocals, that is, their product is unity. These terms apply to specific materials tested between two specified isothermal surfaces. For this practice, materials are considered homogeneous when the value of the thermal conductivity or thermal resistivity is not significantly affected by variations in the thickness or area of the sample within the normally used range of those variables.

3.4 *Thermal Transmission Property Equations for Convective Boundary Conditions*:

3.4.1 *Surface resistance*, R_i, the quantity determined by the temperature difference at steady-state between an isothermal surface and its surrounding air that induces a unit heat flow per unit area to or from the surface. Typically, this parameter includes the combined effects of conduction, convection, and radiation. Surface resistances are calculated as follows:

$$R_h = \frac{A\,(T_1 - T_h)}{Q} \tag{7}$$

$$R_c = \frac{A\,(T_c - T_2)}{Q} \tag{8}$$

NOTE 4—Subscripts 1 and 2 are used to differentiate between the hot and cold side air, respectively.

3.4.2 *Surface coefficient*, h, is often called the film coefficient. These coefficients are calculated as follows:

$$h_h = \frac{Q}{A\,(T_1 - T_h)} \tag{9}$$

$$h_c = \frac{Q}{A\,(T_c - T_2)} \tag{10}$$

NOTE 5—The *surface coefficient, h_i*, and the *corresponding surface resistance, R_i*, are reciprocals, that is, their product is unity.

3.4.3 *Overall thermal resistance, R_u*—the quantity determined by the temperature difference at steady-state between the air temperatures on the two sides of a body or assembly that induces a unit time rate of heat flow per unit area through the body. It is the sum of the resistances of the body or assembly and of the two surface resistances and may be calculated as follows:

$$R_u = \frac{A\,(T_1 - T_2)}{Q} \tag{11}$$
$$= R_c + R + R_h$$

3.4.4 *Thermal transmittance, U* (sometimes called overall coefficient of heat transfer), is calculated as follows:

$$U = \frac{Q}{A\,(T_1 - T_2)} \tag{12}$$

The transmittance can be calculated from the thermal conductance and the surface coefficients as follows:

$$1/U = (1/h_h) + (1/C) + (1/h_c) \quad (13)$$

NOTE 6—Thermal transmittance, *U*, and the corresponding overall thermal resistance, R_u, are reciprocals, that is, their product is unity.

4. Significance and Use

4.1 ASTM thermal test methods are complex because of added apparatus details necessary to ensure accurate results. As a result, many users find it difficult to locate the data reduction details necessary to reduce the data obtained from these tests. This practice is designed to be referenced in the thermal test methods, thus allowing them to concentrate on experimental details rather than data reduction.

4.2 This practice is intended to provide the user with a uniform procedure for calculating the thermal transmission properties of a material or system from standard test methods used to determine heat flux and surface temperatures. This practice is intended to replace the similar calculation sections of Test Methods C 177, C 236, C 335, C 518, C 745, C 976, C 1033, and C 1114.

4.3 This practice provides the method for developing the thermal conductivity as a function of temperature for a specimen from data taken at small or large temperature differences. This relationship can be used to characterize material for comparison to material specifications and for use in calculations programs such as Practice C 680.

4.4 Two general solutions to the problem of establishing thermal transmission properties for application to end-use conditions are outlined in Practice C 1058. (Practice C 1058 should be reviewed prior to use of this practice.) One is to measure each product for each end-use condition. This solution is rather straightforward and needs no other elaboration. The second is to measure each product over the entire temperature range of application conditions and to use these data to establish the thermal-transmission property dependencies on the various end-use conditions. One advantage of the second approach is that once these dependencies have been established, they serve as the basis for estimating the performance for a given product to other conditions.

4.5 **Precaution—**The use of thermal curves developed in Section 6 must be limited to a temperature range that does not extend to less than the lowest surface temperature or higher than the highest surface temperature for the test data set used to generate the curves.

5. Determination of Thermal Transmission Properties for a Specific Temperature

5.1 Using the appropriate test method of interest, determine the steady-state heat flux and temperature data for the test.

NOTE 7—The calculation of thermal properties requires that: (1) the thermal insulation specimen is homogeneous, as defined in Terminology C 168 or, as a minimum, appears uniform across the test area; (2) the measurements are taken only after steady-state has been established; (3) the heat flows in a direction normal to the isothermal surfaces of the specimen; (4) the rate of flow of heat is known; (5) the specimen dimensions, that is, heat flow path length parallel to heat flow, and area perpendicular to heat flow, are known; and (6) both specimen surface temperatures (and equivalently, the temperature difference across the specimen) are known. In the case of a hot box systems test, both air curtain temperatures must be known.

5.2 Choose the thermal test parameter (λ or r', *R* or *C*, *U* or R_u) to be calculated from the test results.

5.3 Calculate the thermal property of interest using the data from the test as described in 5.1, and the appropriate equation in 3.3 or 3.4.

5.4 Using the data from the test as described in 5.1, determine the test mean temperature for the thermal property of 5.3 using the following equation:

$$T_m = (T_h + T_c)/2 \quad (14)$$

NOTE 8—The thermal transmission properties determined in 5.3 are applicable only for the conditions of the test. Further analysis is required using data from multiple tests if the relationship for the thermal property variation with mean test temperature is to be determined. If this relationship is required, the analysis to be followed is presented in Section 6.

5.5 *An Example of a Computation of Thermal Conductivity Measured in a Two-Sided Guarded Hot Plate*:

5.5.1 For a guarded hot plate apparatus in the normal, double-sided mode of operation, the heat developed in the metered area heater passes through two specimens. To reflect this fact, Eq 3 for the operational definition of the mean thermal conductivity of the pair of specimens must be modified to read:

$$\lambda_{exp} = \frac{Q}{A\,[(\Delta T/L)_1 + (\Delta T/L)_2]} \quad (15)$$

where:

$(\Delta/T_{ss}/L)_1$ = the ratio of surface to surface temperature difference to thickness for Specimen 1. A similar expression is used for Specimen 2.

5.5.2 In many experimental situations, the two temperature differences are very nearly equal (within well under 1 %), and the two thicknesses are also nearly equal (within 1 %), so that Eq 15 may be well approximated by a simpler form:

$$\lambda_{exp} = \frac{Q\,L_{average}}{2A\,\Delta T_{average}} \quad (16)$$

where:

$\Delta T_{average}$ = the arithmetic mean temperature difference $(\Delta T_1 + \Delta T_2)/2$,

$L_{average}$ = $(L_1 + L_2)/2$ is the arithmetic mean of the two specimen thicknesses, and

$2A$ = occurs because the metered power flows out through two surfaces of the metered area for this apparatus. For clarity in later discussions, use of this simpler form, Eq 16, will be assumed.

NOTE 9—The mean thermal conductivity, λ_m, is usually not the same as the thermal conductivity, λ (T_m), at the mean temperature T_m. The mean thermal conductivity, λ_m, and the thermal conductivity at the mean temperature, λ (T_m), are equal only in the special case where λ (*T*) is a constant or linear function of temperature **(1)**, that is, when there is no curvature (nonlinearity) in the conductivity-temperature relation. In all other cases, the conductivity, λ_{exp}, as determined by Eq 3 is not simply a function of mean temperature, but depends on the values of both T_h and T_c. This is the reason the experimental value, λ_{exp}, of thermal conductivity for a large temperature difference is not, in general, the same as that for a small difference at the same mean temperature. The discrepancy between the mean conductivity and the conductivity at the mean temperature increases as ΔT increases. Treatment of these differences is discussed in Section 6.

5.5.3 When ΔT is so large that the mean (experimental) thermal conductivity differs from the thermal conductivity at the mean specimen temperature by more than 1 %, the derived thermal conductivity (Eq 3) shall be identified as a mean value, λ_m, over the range from T_c to T_h. Reference **(1)** describes a method for establishing the actual λ versus T dependency from mean thermal conductivity measurements. Proofs of the above statements, along with some illustrative examples, are given in Appendix X3.

6. Determination of the Thermal Conductivity for a Temperature Range

6.1 Consult Practice C 1058 for the selection of appropriate test temperatures. Using the appropriate test method of interest, determine the steady-state heat flux and temperature data for each test covering the temperature range of interest.

6.2 *Small temperature differences*—The use of Eq 15 or Eq 16 is valid for determining the thermal conductivity versus mean temperature only if the temperature difference between the hot and cold surfaces is small. For the purpose of this practice, experience with most insulation materials shows that the maximum ΔT should be 25 K or 5 % of the absolute mean temperature, whichever is greater. The procedure given in 6.2.1 may be followed only when this temperature difference standard is met. The procedure of 6.3 may be used for all test data reduction.

6.2.1 The quantities on the right-hand side of Eq 15 are known for each data point; from these quantities $\lambda(T)$ may be calculated if ΔT is sufficiently small (see 6.2), for normal insulation applications. The value of $\lambda(T)$ so obtained is an approximation, its accuracy depends on the curvature (nonlinearity) of the thermal conductivity-temperature relationship **(1)**. It is conventional to associate the value of λ_{exp} obtained from Eq 16 with the mean temperature T_m at the given data point. For data obtained at a number of mean temperatures, a functional dependence of λ with T may be obtained, with functional coefficients to be determined from the data. For the situation where the number of data points is greater than the number of coefficients in the function, the quantities calculated from the right-hand side over determine the coefficients, and a least-squares fit to the data may be used to obtain the functional dependence of λ with T. The accuracy of the coefficients thus obtained depend not only on the experimental imprecision, but also on the extent to which the thermal conductivity-temperature relationship departs from a linear one over the temperature range defined by the isothermal boundaries of the specimen during the tests.

6.3 *Computation of thermal conductivity when temperature Differences are large*—The following sections apply to all testing results and are specifically required when the temperature difference is greater than about 5 % of the absolute mean temperature. This situation typically occurs during measurements of heat transmission in pipe insulation, Test Method C 335, but may also occur with measurements using other apparatus. Eq 17 and Eq 18 are developed in Appendix X2, but are presented here for continuity of this practice.

6.3.1 The dependence of λ on T for flat-slab geometry is:

$$\lambda_m = \frac{1}{\Delta T}\int_{T_c}^{T_h} \lambda(T)\, dT$$

or:

$$\lambda_m = QL / [2\, A\, (T_h - T_c)] \tag{17}$$

The quantities T_h, T_c, Q, and $(L/2A)$ on the right-hand side are known for each data point obtained by the user.

6.3.2 The dependence of λ on T for cylindrical geometry is:

$$\lambda_m = \frac{1}{\Delta T}\int_{T_c}^{T_h} \lambda(T)\, dT$$

or:

$$\lambda_m = \frac{Q \ln (r_2/r_1)}{2\,\pi\, l\, (T_h - T_c)} \tag{18}$$

6.4 *Thermal conductivity integral (TCI) method*—To obtain the dependence of thermal conductivity on temperature from Eq 17 or Eq 18, a specific functional dependence to represent the conductivity-temperature relation must first be chosen. After the form of the thermal conductivity equation is chosen, 6.4.1-6.4.3 are followed to determine the coefficients for that equation.

6.4.1 Integrate the selected thermal conductivity function with respect to temperature. For example, if the selected function $\lambda(T)$ were a polynomial function of the form

$$\lambda(T) = A_o + A_n T^n + A_m T^m, \tag{19}$$

then, from Eq 18, the temperature-averaged thermal conductivity would be:

$$\lambda_m = \frac{A_o(T_h - T_c)}{(T_h - T_c)} + \frac{A_n\,(T_h^{n+1} - T_c^{n+1})}{(n+1)\,(T_h - T_c)} + \frac{A_m\,(T_h^{m+1} - T_c^{m+1})}{(m+1)\,(T_h - T_c)} \tag{20}$$

6.4.2 By means of any standard least-squares fitting routine, the right-hand side of Eq 20 is fitted against the values of experimental thermal conductivity, λ_{exp}. This fit determines the coefficients in the thermal conductivity function, Eq 19 in this case.

6.4.3 Use the coefficients obtained in 6.4.2 to describe the assumed thermal conductivity function, Eq 19. Each data point is then conventionally plotted at the corresponding mean specimen temperature. When the function is plotted, it may not pass exactly through the data points. This is because each data point represents mean conductivity, λ_m, and this is not equal to the value of the thermal conductivity, $\lambda(T_m)$, at the mean temperature. The offset between a data point and the fitted curve depends on the size of ΔT and on the nonlinearity of the thermal conductivity function.

NOTE 10—Many equation forms other than Eq 19 can be used to represent the thermal conductivity function. If possible, the equation chosen to represent the thermal conductivity versus temperature relationship should be easily integrable with respect to temperature. However, in some instances it may be desirable to choose a form for $\lambda(T)$ that is not easily integrable. Such equations may be found to fit the data over a much wider range of temperature. Also, the user is not restricted to the use of polynomial equations to represent $\lambda(T)$, but only to equation forms that can be integrated either analytically or numerically. In cases where direct integration is not possible, one can carry out the same procedure using numerical integration.

6.5 *TCI method—A summary*—The thermal conductivity integral method of analysis is summarized in the following steps:

6.5.1 Measure several sets of λ_{exp}, T_h, and T_c over a range of temperatures.

6.5.2 Select a functional form for λ(*T*) as in Eq 19, and integrate it with respect to temperature to obtain the equivalent of Eq 20.

6.5.3 Perform a least-squares fit to the experimental data of the integral of the functional form obtained in 6.5.2 to obtain the best values of the coefficients.

6.5.4 Use these coefficients to complete the λ(*T*) equation as defined in 6.5.2. Remember that the thermal conductivity equation derived herein is good only over the range of temperatures encompassed by the test data. Extrapolation of the test results to a temperature range not covered by the data is not acceptable.

7. Consideration of Test Result Significance

7.1 A final step in the analysis and reporting of test results requires that the data be reviewed for significance and accuracy. The following areas should be considered in the evaluation of the test results produced using a Practice C 1045 analysis.

7.2 *Assessment of apparatus uncertainty*—The determination of apparatus uncertainty should be performed as required by the appropriate apparatus test method.

7.3 *Material inhomogeneity*—The uncertainty caused by specimen inhomogeneity can seriously alter the measured dependencies. To establish the possible consequences of material inhomogeneity on the interpretation of the results, the user shall measure an adequate fraction of the product over the entire range of product manufacture variations. The specimen area is generally much smaller than the surface area of the delivered product. If possible, several specimens should be measured to sample a sufficient portion of the product. The resultant mean value of the measurements is representative of the product to within the uncertainty of the mean, while the range of the results is indicative of the product's inhomogeneity. Additional information regarding sampling procedures can be found in Practice E 122.

7.4 *Test grid*—The thermal transmission properties of insulations are dependent on several variables, including product classification, temperature, density, plate emittance, fill-gas pressure, temperature difference, and fill-gas species. A total characterization of these dependencies requires the measurement of the thermal transmission for all combinations of these variables at an adequate number of test points. However, since the producer and consumer of a product are seldom interested in the entire range of properties possible, industry specifications generally require specific test conditions on representative samples. It must be emphasized that the effect of material variability (inhomogeneity) is an important parameter in assessing the significance of results from the test grid.

7.5 *Range of test temperatures*—The test temperature range for each variable shall include the entire range of application to avoid extrapolation of any measured dependency. Guidance for this selection is presented in Practice C 1058.

8. Report

8.1 The report of thermal transmission properties shall include all necessary items specified by the test method followed.

8.2 The total uncertainty of the thermal transmission properties shall be calculated in accordance with the test method and reported.

8.3 The report shall include any test conditions on which the thermal transmission properties are dependent.

8.4 Any corrections shall be specified, and the basis of these corrections shall be given.

8.5 If mean values are reported for tests employing large temperature differences (larger than 5 % of the absolute temperature), the range of temperature differences shall be reported.

8.6 When the thermal conductivity versus temperature relationship has been determined, report the equation with its coefficients, the method of data analysis and regression, and the range of temperatures that were used to determine the coefficients.

8.7 The temperature range of usefulness for the equation coefficients shall be specified.

8.8 Results shall be reported in either SI or conventional units as specified by the requester. If not specified, the results shall be reported in SI units.

9. Keywords

9.1 calculation; integral method; thermal conductance; thermal conductivity; thermal properties; thermal resistance; thermal resistivity; thermal transmission

APPENDIXES

(Nonmandatory Information)

X1. GENERAL DISCUSSION OF THERMAL PROPERTIES MEASUREMENT

X1.1 Thermal transmission properties, that is, thermal conductivity and thermal resistivity, are considered to be intrinsic characteristics of a material. These intrinsic properties are dependent on temperature as well as the microscopic structure of the material. Furthermore, some external influences, such as pressure, may affect the structure of a material and, therefore, its thermal properties. For heterogeneous materials such as those composed of granules, fibers, or foams, additional conductive dependencies arise due to the presence of the fill-gas. As long as the heat flux mechanism is conductive, each of the dependencies is characteristic of the structure and constituents of the material. When only conductive heat flux is present, the measurement, calculation of thermal properties, and application of the results to end-use conditions is well defined by the literature **(1-14)**.

X1.2 In some materials, nonconductive heat fluxes are present that result in property dependencies on specimen dimensions, test temperature conditions, or apparatus parameters. This is not to be confused with the effect of measurement errors that are dependent on specimen or apparatus characteristics. The thermal conductivity of very pure metals at low temperature, for example, actually is dependent on the dimensions of the specimen when they are sufficiently small. This phenomenon is referred to as the size effect, and represents a deviation from conductive behavior. A similar phenomenon occurs in materials that are not totally opaque to radiation. The thermal transmission properties for such materials will be dependent on the specimen thickness and the test apparatus surface plate emittance. This is commonly referred to as the "thickness effect" **(3-9)**. The heat flux in a heterogeneous material containing a fill-gas or fluid may, under certain conditions of porosity or temperature gradient, have a convective heat flux component **(10,11)**. The resulting thermal transmission properties may exhibit dependencies on specimen size, geometry, orientation, and temperature difference.

X1.3 The existence of such nonintrinsic dependencies has caused considerable discussion regarding the utility of thermal transmission properties **(12,13)**. From a practical standpoint, they are useful properties for two reasons. First, the transition from conductive to nonconductive behavior is a gradual and not an abrupt transition, and the dependencies on specimen size, geometry, and orientation are generally small. Second, pseudothermal transmission properties can be calculated that apply to a restricted range of test conditions and are usually denoted by adding the modifier effective or apparent, for example, apparent thermal conductivity. For these pseudo properties to be useful, care must be exercised to specify the range of test conditions under which they are obtained.

X1.4 Some of the thermal-transmission property dependencies of interest may be quite small. It is important that the uncertainties associated with the measurement procedure and material variability are known. Uncertainties caused by systematic errors can seriously alter the conclusions based on the measured dependencies. This point was clearly illustrated by an interlaboratory study on low-density fibrous glass insulation **(14)**. In this round robin, each of five laboratories determined the thickness effect from 2.54 to 10.2 cm thickness. The lowest thickness effect observed was 2 %, while the highest was 6 %. The best estimate of the actual thickness effect clearly involves an in-depth analysis of the measurement errors of each laboratory.

X2. DEVELOPMENT OF EQUATIONS FOR PRACTICE C 1045 ANALYSIS

X2.1 This development of equations necessary to support Practice C 1045 applies to the flow of heat through a homogeneous insulation exhibiting a thermal conductivity that depends on temperature. Existing methods of measurement of thermal conductivity account for various modes of heat transmission, that is, thermal conduction, convection and radiation, occurring within insulation under steady-state, one-dimensional heat flow conditions. Fourier's law of heat conduction has been derived in many heat transfer texts. Fourier's law is generally stated as the heat flux being proportional to the temperature gradient, or:

$$q = -\lambda(T)(dT/dp) \tag{X2.1}$$

where the proportionality coefficient is the thermal conductivity as a function of temperature and p is the coordinate along which heat is flowing. Development of equations for test method heat flow in the slab (Test Methods C 177, C 518, C 1114, etc.) and radial heat flow in the hollow right circular cylinder (Test Method C 335) will be performed using the boundary conditions:

$$T = T_1 \text{ at } x = x_1\text{, or } r = r_1 \tag{X2.2}$$
$$T = T_2 \text{ at } x = x_2\text{, or } r = r_2$$

X2.2 Case 1, Slab Insulation, substituting $p = x$ in Eq X2.1 and performing the indicated integration:

$$q \int_{x_1}^{x_2} dx = -\int_{T_1}^{T_2} \lambda\,(T)\,dt \tag{X2.3}$$

yields:

$$q = \lambda_{eff} \frac{(T_1 - T_2)}{(x_1 - x_2)} \tag{X2.4}$$

where:

$$\lambda_{\text{eff}} = \int_{T_1}^{T_2} \lambda\,(T)\,dt \,/\, (T_1 - T_2) \tag{X2.5}$$

X2.3 Radial heat flow in hollow cylinders, substituting $p = r$ in Eq X2.1, and letting:

$$q = \frac{Q}{2\,\pi\,r\,l} \tag{X2.6}$$

and:

$$q = -\lambda\,(T)\frac{dt}{dr}$$

combining these two expressions and performing the indicated integration:

$$\frac{\mathrm{Q}}{2\,\pi\,l}\int_{\mathrm{r_1}}^{\mathrm{r_2}}\frac{\mathrm{dr}}{\mathrm{r}} = -\int_{\mathrm{T_1}}^{\mathrm{T_2}} \lambda\,(\mathrm{T})\,\mathrm{dT} \tag{X2.7}$$

where:
r = radius.

$$Q = \lambda_{\text{eff}}\frac{2\,\pi\,l\,(T_1 - T_2)}{\ln\,(r_2/r_1)} \tag{X2.8}$$

X3. THERMAL CONDUCTIVITY VARIATIONS WITH MEAN TEMPERATURE

X3.1 The purpose of this appendix is to expand upon statements made in the body of this practice relative to the handling of data from the thermal conductivity tests. Some examples are given to clarify the difference between the analysis of thermal conductivity data taken at large temperature differences and the analysis of conductivity data taken at small temperature differences. The necessity for a difference in analysis method is based on the distinction between mean thermal conductivity, λ_m, and thermal conductivity at the mean temperature, $\lambda(T_m)$, when the conductivity varies nonlinearly with temperature. For this discussion, the arithmetic mean of a variable x is denoted by the subscript m.

X3.2 Eq X3.1 provides the mathematical definition of the mean value of the thermal conductivity with respect to temperature over the range of temperature from T_c to T_h:

$$\lambda_m = \frac{1}{(T_h - T_c)} \int_{T_c}^{T_h} \lambda(T)\, dT \tag{X3.1}$$

X3.3 *Example 1—Thermal Conductivity as a Polynomial Function*:

X3.3.1 The thermal conductivity versus temperature relationship for many typical insulation materials can be defined by a third order polynomial equation. Eq X3.2 describes this thermal conductivity correlation.

$$\lambda(T) = a_0 + a_1 T + a_2 T^2 + a_3 T^3 \tag{X3.2}$$

NOTE X3.1—In this and succeeding examples, the coefficients a_i (i = 0,1,2,...) are constants.

X3.3.2 Substituting Eq X3.2 into Eq X3.1 and integrating the thermal conductivity correlation over temperature, yields:

$$\lambda_m = \frac{[a_0(T_h - T_c) + a_1(T_h^2 - T_c^2)/2 + a_2(T_h^3 - T_c^3)/3 + a_3(T_h^4 - T_c^4)/4]}{(T_h - T_c)} \tag{X3.3}$$

$$= a_0 + a_1(T_h + T_c)/2 + a_2(T_h^2 + T_hT_c + T_c^2)/3 + a_3(T_h^2 + T_c^2)(T_h + T_c)/4$$

X3.3.3 The difference between mean thermal conductivity, λ_m, and thermal conductivity at the mean temperature, $\lambda(T_m)$, as defined by Eq X3.2 and X3.3 yields:

$$\lambda_m - \lambda(T_m) = (T_h - T_c)^2 [a_2/12 + a_3/8 (T_h + T_c)] \tag{X3.4}$$

X3.3.4 Eq X3.4 shows that this difference between the mean thermal conductivity and the thermal conductivity at the mean temperature is zero, regardless of the values of the constants a_0 and a_1, and is therefore valid also for the special cases (1) constant thermal conductivity with temperature ($\lambda = a_0$ = constant, that is, the terms a_1, a_2, and a_3 are zero), and (2) linear thermal conductivity ($\lambda = a_0 + a_1T$, that is, a_2 and a_3 are zero), as well.

X3.3.5 Eq X3.4 also shows that for materials where the coefficients a_2 and a_3 are not zero, the difference between the mean thermal conductivity and the thermal conductivity at the mean temperature is a strong function of the temperature difference, $(T_h - T_c)$.

X3.4 *Example 2—"Real" Data*

X3.4.1 The final example illustrates the magnitude of the difference, $\lambda_m - \lambda(T_m)$, based on data for temperatures ranging from 13 to 435°C for fibrous board insulation. This data, presented in Table X3.1, were acquired from measurements on the same specimen set at both limited (ΔT < 110K) and variable (ΔT up to 360 K) temperature differences. The fibrous board insulation has been represented by an equation of the form:

$$\lambda(T) = a_0 + a_1 T + a_3 T^3 \tag{X3.5}$$

X3.4.2 Combining Eq X3.1 and Eq X3.5, the equation for λ_m becomes:

$$\lambda_m = a_0 + a_1(T_h + T_c)/2 + a_3(T_h^2 + T_c^2)(T_h + T_c)/4 \tag{X3.6}$$

X3.4.3 Using the data in Table X3.1 and Eq X3.6 and solving for coefficients of X3.6 using a standard statistical analysis program yields the following values for the coefficients for the fibrous board insulation described by Eq X3.5:

$$a_0 = 0.03044 \tag{X3.7}$$

$$a_1 = 1.287E{-}04 \tag{X3.8}$$

$$a_3 = 1.078E{-}09 \tag{X3.9}$$

where the temperatures are in degrees celsius. Note that the standard estimate of error provided by the spreadsheet analysis for the correlation of this data below was 0.0003 W/mK

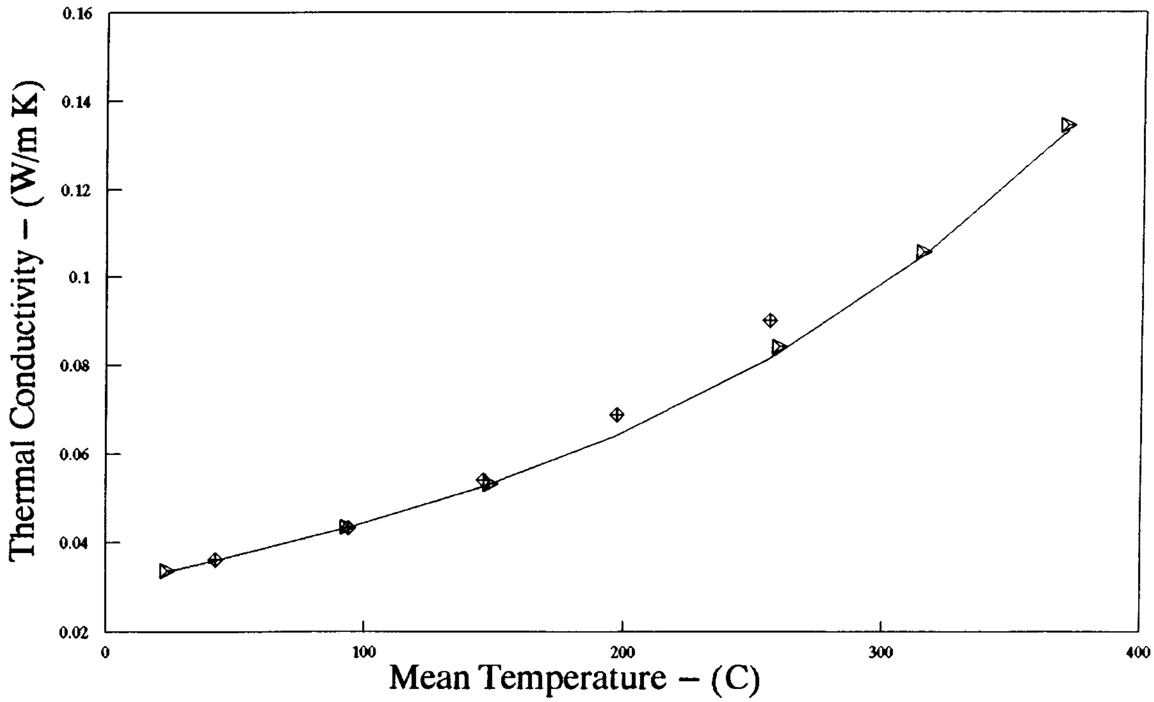
X3.4.4 Fig. X3.1 shows how the test data compares with the final data regression equation as a function of mean temperature. Notice that as the temperature difference of the data increase, the difference between the regression line and data points increases.

X3.5 *Summary*:

X3.5.1 The above examples reveal the method by which one can obtain $\lambda(T)$ from measurements of at large temperature differences. The method described here is referred to as the integral method and is described in detail in Ref **(12)**. First, note that any experimental value of thermal conductivity, λ_{exp}, obtained using Eq 2 and measured values of q, T, and L, is really a value of λ_m and not $\lambda(T_m)$. In addition, note that values of T_h and T_c are available from the experiment. Therefore all of the variables in Eq X3.6 except the coefficients have been

TABLE X3.1 Measured Apparent Thermal Conductivity versus Hot and Cold Face Temperatures

Hot Face Temperature (°C)	Cold Face Temperature (°C)	Thermal Conductivity (W/mK)
35.0	12.8	0.0336
60.1	25.2	0.0362
121.1	65.6	0.0436
154.6	33.3	0.0434
176.1	121.1	0.0531
247.3	44.4	0.0539
315.6	204.4	0.0841
336.4	58.2	0.0687
371.1	260.0	0.1057
426.7	315.6	0.1346
434.5	77.4	0.0900



NOTE—______ = thermal conductivity curve; ◇ = variable temperature difference; ▷ = limited temperature difference.

FIG. X3.1 Comparison of Test Analysis Results

experimentally determined. If the experiment is repeated over a range of values of T_h and T_c, the entire data set can be used to evaluate the best values of the coefficients by normal least-squares fitting procedures. Once these coefficients have been determined, they are equally applicable to Eq X3.5, and $\lambda(T)$ is therefore known.

NOTE X3.2—This procedure is based on the assumption that a unique dependence of thermal conductivity on temperature exists for the material. Such a unique dependence may only be approximate, depending on the coupling effects of the underlying heat transfer mechanisms or irreversible changes in the material during the measurement process. The most convenient check to determine the existence of such effects is to intermix data of both small and large temperature differences in the fit of Eq X3.11. If the deviations of these data from values calculated from Eq X3.11 are systematically dependent on the temperature difference, two possibilities must be considered: (1) a unique temperature dependence does not exist and the systematic dependence on temperature difference is a measure of this inconsistency; or (2) the apparatus or measurement procedure produces a systematic bias that depends on temperature difference. To determine which of the two possibilities is the cause of the indicated inconsistency, detailed examination of the apparatus and procedure, along with further experimentation, is necessary.

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